Understanding the Uses of Numbers

Chemistry is a quantitative science. Theories are based on and supported by measurements and calculations. Most chemistry experiments involve not only measuring but also a search for the meaning of the measurements. Chemistry students are required to learn how to interpret as well as perform calculations using these measurements.

This part of the skills book provides a brief introduction to understanding the uses of numbers. As you continue your education, you will delve into these topics in greater depth. For now, however, we will review topics you have probably already covered in previous science classes.

MEASUREMENT AND THE METRIC SYSTEM (Orange book, Section 3.2 Pg 73-78)

Metric units were first introduced in France more than 100 years ago. A modernized form of the metric system was internationally adopted in 1960. The system is called "SI," which is an abbreviation of its French name, *Le Système International d'Unités.* SI units are used by scientists in all nations, including the United States. This system has a small number of base units from which all other necessary units are derived.

(orange book, Table 3.1 Pg 73)

Quantity	Unit	Abbreviation
Mass	kilogram	kg
Length	meter	m
Time	second	s
Temperature	kelvin	K
Amount of substance	mole	mol
Electric current	ampere	A

[Don't need to memorize

Figure 1 *Units of Measurement*

The limited number of units in Figure 1 is not sufficient for every type of measurement a chemist might need to make. For example, the SI unit of length is the meter (symbolized by m). Most doorways are about two meters high. However, many lengths we may wish to measure are either much larger (distance from the earth to the sun) or much smaller (width of a dime) than a meter. To handle such measurements easily, common metric prefixes are used to change the size of the unit. The distance from the earth to the sun can be expressed in (orange book, Table 3.2 & 3.3) Pg 74

kilometers (km), and the width of a dime can be expressed in terms of millimeters (mm). Figure 2 lists the most useful metric prefixes and their meanings.

[Don't need to memorize mega, all others you do]

Prefix	Abbreviation	Meaning	Example
mega-	M	10^6	1 megabyte = 1 000 000 bytes
kilo-	k	10^3	1 kilogram = 1000 grams
deci-	d	10^{-1}	1 deciliter = 0.1 L
centi-	c	10^{-2}	1 centimeter = 0.01 m
milli-	m	10^{-3}	1 milliampere = 0.001 A
micro-	μ	10^{-6}	1 micrometer = 10^{-6} m
nano-	n	10^{-9}	1 nanometer = 10^{-9} m
pico-	p	10^{-12}	1 picometer = 10^{-12} m

Alternate method

$1\ Mm = 1 \times 10^6 m$
$1\ km = 1 \times 10^3 m$
$1 \times 10^1 dm = 1m$
$1 \times 10^2 cm = 1m$
$1 \times 10^3 mm = 1m$
$1 \times 10^6 \mu m = 1m$
$1 \times 10^9 nm = 1m$
$1 \times 10^{12} pm = 1m$

Figure 2 *Common Metric Prefixes*

Quantities can be converted from one unit to another through the use of equivalences from Figure 2 and a unit conversion factor.

Example 1: Convert 1456 g to kilograms.

Equivalence: 1 kilogram = 1×10^3 g

From the equivalence we create a **unit conversion factor.** This is a fraction in which the numerator is a quantity that is equal to the quantity of the denominator—except the numerator is expressed in different units. A unit conversion factor is equal to one.

Conversion factor: $\dfrac{1\ kilogram}{1 \times 10^3\ gram}$ or $\dfrac{1 \times 10^3\ gram}{1\ kilogram}$

The conversion factor is chosen to cause the original unit to cancel and the desired unit to remain.

Multiplication: $1456\ \cancel{g} \times \dfrac{1\ kg}{1 \times 10^3\ \cancel{g}} = 1.456\ kg$

More than one conversion factor can be used in a single problem.

Example 2: Convert 325 mg to kilograms.

Equivalences: $1\ g = 1 \times 10^3$ mg

$1\ kg = 1 \times 10^3$ g

Conversion factors: $\dfrac{1\ g}{1 \times 10^3\ mg}$ *and* $\dfrac{1\ kg}{1 \times 10^3\ g}$

Multiplication: $325\ mg \times \frac{1\ g}{1 \times 10^3\ mg} \times \frac{1\ kg}{1 \times 10^3\ g} \times 3.25 \times 10^{-4}\ kg$

Converting units in this manner is called **dimensional analysis or factor labeling.** Dimensional analysis can be used to solve many different types of problems in chemistry. For further instruction on dimensional analysis, see the next section.

Remember that all numbers in chemistry are an outcome of a measurement. As a result numbers should have a measurement unit associated with them. Always include units when you write numbers.

Practice Problems

1. Which metric unit and prefix would be most convenient to measure each of the following?
   a. the diameter of a giant sequoia tree
   b. the diameter of a human hair
   c. time necessary to blink your eye
   d. mass of gasoline in a gallon
   e. mass of a cold virus
   f. amount of aspirin in a tablet
   g. mass of concrete to pave a parking lot
2. What word prefixes are used in the metric system to indicate the following multipliers?
   a. 1×10^3 b. 1×10^{-3} c. 0.01 d. 1×10^{-6}
3. An antacid tablet contains 168 mg of the active ingredient ranitidine hydrochloride. How many grams of the compound are in the tablet?
4. There are 1.609 km in 1 mile. Determine the number of centimeters in a mile.
5. A paper clip is 3.2 cm long. What is its length in millimeters?
6. State at least one advantage of SI units over the customary US units.

Frequently we wish to measure quantities that cannot be expressed using one of the basic SI units. In these situations two or more units are combined to create a new unit. These units are called **derived units.** For example, speed is defined as the ratio of distance to time. To measure speed, two units—distance and time—are combined. Another derived unit frequently used in the laboratory is volume. Let us see how these derived units are related to SI units.

The volume of a cube is determined by multiplying (length × width × height). A cube with sides of 10 cm × 10 cm × 10 cm has a volume of 1000 cm^3, which is defined as a liter.

$1000\ cm^3 = 1$ liter (1 L)

Density is the ratio of mass to volume $\left(D = \frac{M}{V}\right)$, so it is also a derived unit. It is an important property for determining the identity of a sample of matter. Again, two units are combined—a mass unit and a volume unit. The density of solids and liquids is usually expressed as g/cm^3 and the density of gases as g/L.

Practice Problems

1. The average person in the United States uses 340 L of water daily. Convert this to milliliters.
2. A quart is approximately equal to 946 mL. How many liters are in 1 quart?
3. One hundred fifty milliliters of rubbing alcohol has a mass of 120 g. What is the density of rubbing alcohol?
4. A ruby has a mass 7.5 g and a volume of 1.9 cm^3. What is the density of this ruby?
5. What is the density of isopropyl alcohol if 5.00 mL weigh 3.93 g?

DIMENSIONAL ANALYSIS (orange book, Section 3.3, pg 80-87)

Dimensional analysis, also called the factor-label method, is widely used by scientists to solve a wide variety of problems. You have already used this method to convert one type of metric unit to another. The method is helpful in setting up problems and also in checking work because if the unit label is incorrect, the numbers in the answer to the problem are also incorrect. The use of dimensional analysis consists of three basic steps:

1. Identify equivalence relationships in order to create unit conversion factors.
2. Identify the given unit and the new unit desired.
3. Arrange the conversion factor so given units cancel, leaving the new desired unit. Perform the calculation.

The following example illustrates the use of dimensional analysis.

Example 1: In an exercise your laboratory partner measured the length of an object to be 12.2 inches. All other measurements were in centimeters and the answer was to be reported in cm^3. Another member of the group could measure the object in centimeters, or 12.2 inches could be converted to centimeters.

Step 1: Find the equivalence relating centimeters and inches.

$$2.54 \text{ cm} = 1 \text{ in}$$

Step 2: Identify the given unit and the "to find" unit

Given unit = inches

"To find" unit = cm

Step 3: Choose the fraction with the "given" quantity in the denominator and the "to find" quantity in the numerator.

$$12.2 \cancel{in} \times \frac{2.54 \, cm}{1 \cancel{in}} = 30.0 \, cm$$

Cancel labels as shown above.

Frequently, more than one conversion factor is necessary to solve a particular problem.

Example 2: How many seconds are in 24 hours?

Step 1: Identify an equivalence:

$$1 \text{ hr} = 60 \text{ min} \qquad 1 \text{ min} = 60 \text{ sec}$$

Step 2: Given unit: hours; "to find" unit: sec

Step 3: Arrange for given unit to cancel and progress to the desired unit:

$$24 \cancel{\text{hr}} \left(\frac{60 \cancel{\text{min}}}{1 \cancel{\text{hr}}}\right)\left(\frac{60 \text{ sec}}{1 \cancel{\text{min}}}\right) = 86\,400 \text{ sec}$$

Practice Problems

1. The distance between New York and San Francisco is 4 741 000 m. Now, that may sound impressive, but to put all those digits on a car odometer is slightly inconvenient. (Of course, in the United States the odometer measures miles, but that is another story.) In this case, kilometers are a better choice for measuring distance. Change the distance to kilometers.
2. Convert 7265 mL to L.
3. The 1500 meter race is sometimes called the "metric mile." Convert 1500 m to miles. (1 m = 39.37 in).
4. The density of aluminum is 2.70 g/cm^3. What is the mass of 235 cm^3 of aluminum?
5. How many 250 mL servings can be poured from a 2.0 L bottle of soft drink?
6. The speed limit in Canada is 100 km/hr. Convert this to meters/second.
7. The density of helium is 0.17 g/L at room temperature. What is the mass of helium in a 5.4 L helium balloon?
8. Liquid bromine has a density of 3.12 g/mL. What volume would 7.5 g of bromine occupy?
9. An irregularly shaped piece of metal has a mass of 147.8 g. It is placed in a graduated cylinder containing 30.0 mL of water. The water level rises to 48.5 mL. What is the density of the metal?

PRECISION AND ACCURACY IN MEASUREMENT (orange book, Section 3.1) Pg 62-72

Chemistry experiments often require a number of different measurements, and there is always some error in measurement. How much error depends on several factors, such as the skill of the experimenter, the quality of the instrument, and the design of the experiment. The reliability of the measurement has two components: precision and accuracy. **Precision** refers to how closely measurements of the same quantity agree. A high-precision measurement is one that produces very nearly the same result each time it is measured. **Accuracy** is how well measurements agree with the accepted or true value.

It is possible for a set of measurements to be precise without being accurate. Figure 3 demonstrates different possible combinations of precision and accuracy in an experiment designed to hit the center of the target.

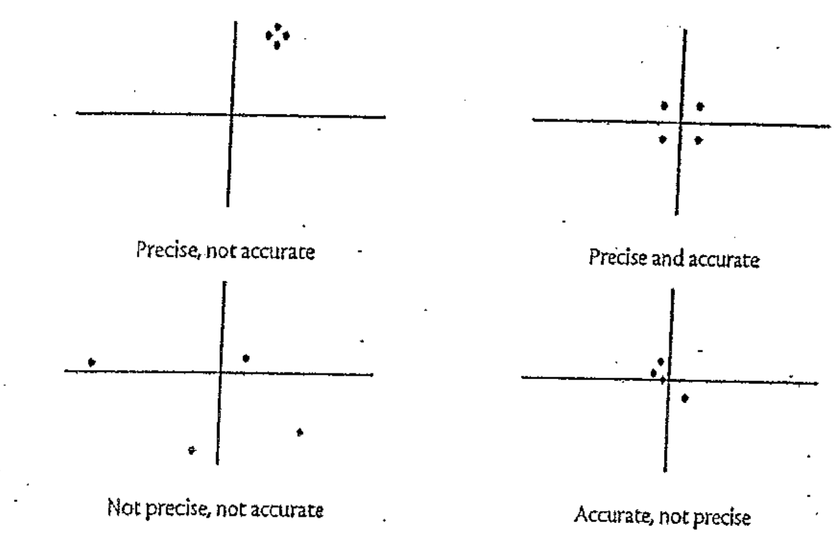


Figure 3 *Precision and Accuracy*

A second example of accuracy and precision is given in Figure 4. The table lists the results of temperature measurements of a beaker of boiling water. The standard temperature of boiling water is 100 °C. The data in the table illustrates the different possible combinations of precision and accuracy in an experiment:

Reading	Thermometer 1	Thermometer 2	Thermometer 3	Thermometer 4
1	99.9 °C	97.5 °C	98.3 °C	97.5 °C
2	100.1 °C	102.3 °C	98.5 °C	99.7 °C
3	100.0 °C	99.7 °C	98.4 °C	96.2 °C
4	99.9 °C	100.9 °C	98.7 °C	94.4 °C
Average	100.0 °C	100.1 °C	98.5 °C	97.0 °C
Range	0.2 °C	4.8 °C	0.4 °C	5.3 °C

Figure 4 *Measured Temperature of 100 mL of Boiling Water*

The average value for each set is taken as the best value. The range—the difference between the largest and smallest values—is the measure of the agreement among the individual measurements. The data taken with Thermometer 1 is accurate and precise, since the average agrees with the accepted value and the range is small. Thermometer 2 provided data that is accurate but not precise since the range is relatively large. The data from Thermometer 3 is precise but not accurate. The range is small enough that it is possible that Thermometer 3 may not have been calibrated properly. Thermometer 4 provides data that is neither precise nor accurate.

Activity

1. Your teacher will provide you with several balances and an object to weigh.
2. Carefully measure the mass of your object 5 times on each balance provided.
3. Calculate the average and the range for each balance. Your teacher will provide you with the "actual" mass of your object.
4. Discuss the accuracy and the precision of each balance you used.
5. Suggest possible errors that might have occurred with the use of each balance.

Practice Problems

1. Determine the precision and accuracy of the following sets of measurements.
   a. A group of students was determining the density of an unknown liquid. They obtained the following values:
   $1.34\,\frac{g}{cm^3}$, $1.32\,\frac{g}{cm^3}$, $1.36\,\frac{g}{cm^3}$. The actual value is $1.34\,\frac{g}{cm^3}$.
   b. Another group obtained the same results, but the actual value is $1.40\,\frac{g}{cm^3}$.
   c. A third group obtained the following values:
   $1.66\,\frac{g}{cm^3}$, $1.28\,\frac{g}{cm^3}$, $1.18\,\frac{g}{cm^3}$. The actual value is $1.34\,\frac{g}{cm^3}$.
   d. A fourth group obtained the following values: $1.60\,\frac{g}{cm^3}$, $1.70\,\frac{g}{cm^3}$, $1.40\,\frac{g}{cm^3}$. The actual value is $1.40\,\frac{g}{cm^3}$.

Percent error is a measurement of the accuracy of the measurement. It is calculated using the following formula:

$$\textit{Percent Error} = \frac{\textit{Experimental value} \times \textit{Accepted value}}{\textit{Accepted value}} \times 100\%$$

NOTE: Percent error is a positive number when the experimental value is too high and is a negative number when the experimental value is too low.

SIGNIFICANT FIGURES (orange book, Section 3.1, pg 62-72)

As discussed earlier, measurements are an integral part of most chemical experimentation. However, the numerical measurements that result have some inherent uncertainty. This uncertainty is a result of the measurement device as well as the fact that a human being makes the measurement. No measurement is absolutely exact. When you use a piece of laboratory equipment, read and record the measurement to one decimal place beyond the smallest marking on the piece of equipment. The length of the red arrow placed along the centimeter stick is 4.75 cm long. There are no graduation markings to help you read the last measurement as 5. This is an estimate. As a result this digit is uncertain. Another person may read this as 4.76 cm. This is acceptable since it is an estimation. There is error (uncertainty) built into each measurement and cannot be avoided.

If the measurement is reported as 4.75 cm, scientists accept the principle that the last digit has an uncertainty of ±0.01 cm. In other words the length might be as small as 4.74 cm or as large as 4.76 cm. It is understood by scientists that the last digit recorded is an estimation and is uncertain. It is important to follow this convention.

Guidelines for Determining Significant Digits

1. **All digits recorded from a laboratory measurement are called significant figures (or digits).**
   The measurement of 4.75 cm has three (3) significant figures.
   NOTE: If you use an electronic piece of equipment, such as a balance, you should record the measurement exactly as it appears on the display.

[Def: All known digits (1-9) and one unknown/guess digit.]

Measurement	Number of Significant Figures
123 g	3
46.54 mL	4
0.33 cm	2
3 300 000 nm	2
0.033 g	2

Figure 5 *Significant Figures*

2. **All non-zero digits are considered significant.**
   There are special rules for zeros. Zeros in a measurement fall into three types: leading zeros, trailing zeros, and middle zeros.

3. **A middle zero is *always* significant.**

   303 mm: a middle zero—always significant

4. **A leading zero is never significant.** It is only a placeholder; not a part of the actual measurement.

   0.0123 kg: a leading zero—never significant

5. **A trailing zero is significant when it is to the right of a decimal point.** This is not a placeholder. It is a part of the actual measurement.

23.20 mL: a trailing zero—significant to the right of a decimal point

6. **All significant figures include units since they are a result of a measurement.** A number without units has little significance.

> The most common errors concerning significant figures are: (1) recording all digits on the calculator readout, (2) failing to include significant trailing zeros (14.150 g), and (3) considering leading zeros to be significant—0.002 g has only one significant figure, not three.

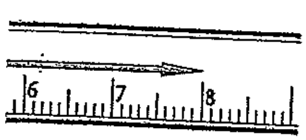
Activity

1. Your teacher will display several different measuring devices. Examine each and determine what digit the last recorded number will occupy.
2. Make the following measurements. Record them using the correct number of significant digits and the measuring device mentioned.
   a. The length of the *ChemCom* book using a meterstick.
   b. The length of the *ChemCom* book using a metric ruler.
   c. The volume of water in a 100-mL graduated cylinder.
   d. The volume of water in a 150-mL beaker.
   e. The volume of water in a 10-mL graduated cylinder.
   f. Measure the mass of a 150-mL beaker.

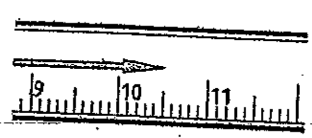
Practice Problems

1. How many significant figures are in each of the following?
   a. 451 000 m
   b. 6.626×10^{-34} J • s
   c. 0.0065 g
   d. 4056 V
   e. 0.0540 mL
2. For the centimeter rulers below record the length of the arrow shown.

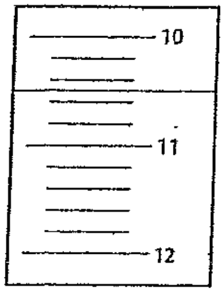
a.

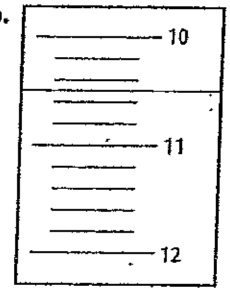


b.



3. The drawings below represent graduated cylinders. Record the volume represented in each picture.

a.


b.


[Sig fig]

USING SIGNIFICANT FIGURES IN CALCULATIONS

Addition and Subtraction

The number of *decimal places* in the answer should be the same as in the measured quantity with the smallest number of *decimal places.*

[Think: Columns]

$$\begin{array}{r} 1259.1\ m \\ 2.365\ m \\ \underline{15.34\ m} \\ 1277.075\ m = 1277.1\ m \end{array}$$

Multiplication and Division

The number of *significant figures* in the answer should be the same as in the measured quantity with the smallest number of *significant figures.*

$$\frac{13.356\ g}{10.42\ mL} = 1.2817658\ g/mL = 1.282\ g/mL$$

[Think: # of sig fig in values of the question/problem]

Activity

Find the volume of the block provided by measuring the length, width, and height. Calculate the volume, showing your work, significant digits, and units.

Practice Problems

1. Answer the following problems using the correct number of significant figures.
   a. $16.27 + 0.463 + 32.1$
   b. $42.05 - 3.6$
   c. 15.1×0.032
   d. $13.36/0.0468$
   e. $\dfrac{(13.36 + 0.046) \times 12.6}{1.424}$
2. In the laboratory a group of students was assigned to determine the density of an unknown liquid. They used a buret to measure the liquid

and found a volume of 2.04 mL. The mass was determined on an analytical balance to be 2.260 g. How should they report the density of the liquid?

3. In the first laboratory activity of the year, students were assigned to find the total area of three tabletops in the room. To save time, each of the three students grabbed a ruler and measured the dimensions. They then calculated the area for each tabletop and added them together. Figure 6 presents the students' measurements. What is the total area of the three tabletops?

Student	Length	Width
A	127 cm	74 cm
B	1.3 m	0.8 m
C	50. in	29.5 in

Figure 6 *Tabletop Dimensions*

NOTE: Only numbers resulting from measurements made using instruments have significant figures and have an infinite number of significant figures. Exact numbers include numbers derived from counting and definition. *Examples:* 25 desks in a room or 100 cm = 1 meter

SCIENTIFIC NOTATION

In chemistry we deal with very small and very large numbers. It is awkward to use many zeros to express very large or very small numbers, so scientific notation is used. The number is rewritten as the product of a number between 1 and 10 and an exponential term—10^n, where n is a whole number.

Examples

1. The distance between New York City and San Francisco = 4 741 000 meters:

$$4\,741\,000 \text{ m} = (4.741 \times 1\,000\,000) \text{ m, or } 4.741 \times 10^6 \text{ m}$$

2. The mass of ranitidine hydrochloride in an antacid tablet = 0.000479 moles:

$$0.000479 \text{ mol} = 4.79 \times 0.0001 \text{ mol, or } 4.79 \times 10^{-4} \text{ mol}$$

It is easier to assess the magnitude and to perform operations with numbers written in scientific notation. It is also easier to identify the proper number of significant figures.

Addition/Subtraction Using Scientific Notation

1. Convert the numbers to the same power of ten.
2. Add (subtract) the nonexponential portion of the numbers.
3. The power of ten remains the same.

Example: $1.00 \times 10^4 + 2.30 \times 10^5$

A good rule to follow is to express all numbers in the problem to the highest power of ten.

Convert 1.00×10^4 to 0.100×10^5.

$0.100 \times 10^5 + 2.30 \times 10^5 = 2.40 \times 10^5$

Multiplication Using Scientific Notation

1. The numbers (including decimals) are multiplied.
2. The exponents are added.
3. The answer is converted to scientific notation—the product of a number between 1 and 10 and an exponential term.

Example: $(4.24 \times 10^2) \times (5.78 \times 10^4)$
$(4.24 \times 5.78) \times (10^{2+4}) = 24.5 \times 10^6$
Convert to scientific notation $= 2.45 \times 10^7$

Division Using Scientific Notation

1. Divide the decimal parts of the number.
2. Subtract the exponents.
3. Express the answer in scientific notation.

Example: $(3.78 \times 10^5) \div (6.2 \times 10^8)$
$(3.78 \div 6.2) \times (10^{5-8}) = 0.61 \times 10^{-3}$
Convert to scientific notation $= 6.1 \times 10^{-4}$

Calculator Use for Scientific Notation Calculations

Most students use calculators to perform operations with exponential numbers. Scientific calculators have a button labeled $\boxed{\text{EXP}}$ or $\boxed{\text{EE}}$ which enters the "10" portion of the number. Apply the following keystrokes to enter the number 4.741×10^6:

$\boxed{4}\ \boxed{\cdot}\ \boxed{7}\ \boxed{4}\ \boxed{1}\ \boxed{\text{EXP}}\ \boxed{6}$

If your answers are consistently incorrect by a power of ten, you are probably entering an extra "10" following the $\boxed{\text{EXP}}$ key. When using a calculator to add or subtract exponential numbers, it is not necessary to first convert the numbers to the same power of ten.

Practice Problems

1. Convert the following numbers to exponential notation.
   a. 0.0000369
   b. 0.0452
   c. 4 520 000
   d. $\frac{36}{1000}$
   e. 365 000
2. Carry out the following operations:
   a. $(1.62 \times 10^3) + (3.4 \times 10^2)$
   b. $(1.75 \times 10^{-1}) - (4.6 \times 10^{-2})$
   c. $(15.1 \times 10^2) \times (3.2 \times 10^{-2})$
   d. $(6.02 \times 10^{23}) \times (2.0 \times 10^2)$
   e. $(6.02 \times 10^{23}) \div (12.0)$
   f. $(6.63 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}) \times \left(3.00 \frac{\mathrm{m}}{\mathrm{s}} \text{ or } \div 4.6 \times 10^{-9} \mathrm{m}\right)$